SOLUTION OF THE EQUATIONS FOR COAGULATION AND RECOMBINATION IN A TURBULENT MEDIUM

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Coagulation and recombination in a turbulent medium is studied statistically with the use of Lagrangian coordinates.

The theoretical study of coagulation and recombination in the presence of turbulent diffusion is a problem of great difficulty. The causes of the difficulty are basically as follows. It is a complicated matter to obtain a closed system of equations for the average (over the turbulent fluctuations) quantities such as the concentration of particles and the spectrum because the effect of large fluctuations, comparable in amplitude to the average quantities themselves. Furthermore, the equations obtained are usually nonlinear partial differential equations which can normally be soved only by resorting to numerical methods. Finally the use of parabolic equations to describe turbulent transport is possible only when the spatial scale is large compared to the characteristic scale of turbulent mixing [1], and the latter is often comparable to the characteristic scale of the problem. For these reasons there is practically no discussion in the literature of coagulation and recombination in the presence of turbulent diffusion.

These difficulties can be overcome to a significant degree by using the Lagrangian approach to describe turbulent mixing [2]. In this approach we follow the evolution of the various quantities of interest along the individual particle trajectories and the solutions are averaged (assuming it is possible to obtain solutions) by taking into account the Lagrangian statistical characteristics of the velocities [1].

We consider the process of coagulation in a uniform, incompressible, turbulent medium, using the Lagrangian approach. The equation for the mass spectrum f(m) of the particles has the form

$$\frac{Df}{Dt} = \frac{1}{2}\int_0^m \beta(m', m-m') f(m') f(m-m') - f(m')\int_0^\infty \beta(m, m') f(m') dm', \tag{1}$$

where D/Dt is the total derivative. Here we use the approximation in which the exchange of particles between turbulent regions is neglected; this implies the condition $n\ell^3_K \ll 1$, where ℓ_K is the Kolmogorov microscale of the turbulence. In the Lagrangian approach (1) is solved for the individual trajectories and so the spatial dependence drops out. The resulting equation has been studied extensively [3].

We assume that the solution of this equation can be found in the form $f = f(\{f_0(m, \mathbf{r}_0)\}, t)$ where $f_0(m, \mathbf{r}_0)$ is the initial spectrum at t = 0, and $\mathbf{r}_0$ is the initial position. Transforming to Euler coordinates, we obtain $f(\mathbf{r}\ m, t) = f(\{f_0(m, \mathbf{r}_0(\mathbf{r}, t))\}, t)$, where $\mathbf{r}_0(\mathbf{r}, t)$, the integral of the equation $d\mathbf{r}/dt = \mathbf{v}(\mathbf{r}, t)$, satisfies the condition

$$\frac{\partial \mathbf{r}_0}{\partial t} + (\mathbf{v}\nabla)\mathbf{r}_0 = 0 \tag{2}$$

with the initial condition $r_0(\mathbf{r}, 0) = \mathbf{r}$. We represent the solution n the form $f = \int\delta(\mathbf{r}_0(\mathbf{r}, t) - \mathbf{r}')\, f(\{f_0(m, \mathbf{r}')\}, t)d\mathbf{r}'$ and average over the turbulent fluctuations (an average is denoted by an overbar). We then obtain

$$\overline{f} = \int G(\mathbf{r}, \mathbf{r}', t) f(\{f_0(m, \mathbf{r}')\}, t) d\mathbf{r}', \tag{3}$$

where the function $G(\mathbf{r}, \mathbf{r}', t) = \overline{\delta(\mathbf{r}_0(\mathbf{r}, t) - \mathbf{r}')} = \overline{\delta\ (\mathbf{r}(t, \mathbf{r}') - \mathbf{r})}$ describes the turbulent diffusion from an instantaneous source at the point $\mathbf{r}'$ of unit intensity. This function has been quite extensively studied, both theoretically and experimentally [1, 4].

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We illustrate the approach for the example of a constant coagulation kernel β = const. In this case the solution of the coagulation equation can be written in terms of the generating function (Laplace transform)

$$\Phi(p, t) = \frac{n^2 \Phi_0(p, \mathbf{r}_0)}{n_0^2(\mathbf{r}_0) + (n - n_0(\mathbf{r}_0)) \Phi_0(p, \mathbf{r}_0)}, \tag{4}$$

where $\Phi_0(p, \mathbf{r}_0)$ is the initial spectrum at t = 0, and $n_0(\mathbf{r}_0)$ is the initial concentration. The dependence of the concentration n(t) on time is given by the law of binary recombinations as

$$n = \frac{n_0(\mathbf{r}_0)}{1 + \frac{\beta}{2} t n_0(\mathbf{r}_0)}. \tag{5}$$

The quantity $\bar{n}$ is then

$$\bar{n} = \int G(\mathbf{r}, \mathbf{r}', t) \frac{n_0(\mathbf{r}')}{1 + \frac{\beta}{2} t n_0(\mathbf{r}')} d\mathbf{r}'. \tag{6}$$

In practice the Gaussian approximation $G(\mathbf{r}, \mathbf{r}', t) \sim \exp(-(\mathbf{r} - \mathbf{r}')^2/2\sigma^2(t))$ is often completely satisfactory. The variance $\sigma^2(t)$ is determined by the Taylor formula in terms of the Lagrangian velocity correlation function [1].

We consider a special case. Let the initial concentration distribution be a Gaussian:

$$n_0(\mathbf{r}_0) = (2\pi\sigma_0^2)^{-3/2} Q \exp\left(-\frac{r_0^2}{2\sigma_0^2}\right). \tag{7}$$

Then

$$\bar{n} = \sum_{k=1}^{\infty} (-1)^{k-1} \left(\frac{\beta t}{2}\right)^{k-1} \left(\frac{Q}{(2\pi\sigma_0^2)^{3/2}}\right)^k \frac{\exp\left(-\frac{r^2}{2(\sigma^2(t) + \sigma_0^2/k)}\right)}{\left(1 + k\frac{\sigma^2(t)}{\sigma_0^2}\right)}. \tag{8}$$

Asymptotically $t \to \infty (\sigma^2(t) \gg \sigma_0^2)$ we have

$$\bar{n} \approx \frac{Q \exp\left(-\frac{r^2}{2\sigma^2(t)}\right)}{(2\pi\sigma^2(t))^{3/2}} \frac{4}{3\sqrt{\pi}} \left(\frac{\beta t}{2} \frac{Q}{(2\pi\sigma_0^2)^{3/2}}\right)^{-1} \ln^{3/2}\left(\frac{\beta t}{2} \frac{Q}{(2\pi\sigma_0^2)^{3/2}}\right).$$

Diffusion determines the decay law $\sim \sigma^3(t)$, coagulation gives the decay law t^{-x} and a combination of these two effects gives the factor $\ln^{3/2} t$, which is obviously due to a decrease in the rate of coagulation.

The evolution of the spectrum is conveniently analyzed in terms of the moments $M_k = \int_0^\infty f(m) m^k dm = (-1)^k d^k/dp^k \, \Phi|_{p=0}$, which in our case satisfy the recursion relation

$$M_k(t) = M_k^0(\mathbf{r}_0) + \frac{\beta t}{2} \sum_{s=1}^{k-1} \frac{k!}{s!(k-s)!} M_s M_{k-s}^0(\mathbf{r}_0). \tag{9}$$

NOTATION

f(m), mass spectrum of the particles; β (m, m'), coagulation kernel; $\mathbf{v}(\mathbf{r}, t)$, turbulent velocity field; $G(\mathbf{r}, \mathbf{r}', t)$, instantaneous source function for turbulent diffusion; $\Phi(p, t)$, generating function of the spectrum; n, concentration of particles.

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STABILITY OF THE LAMINAR BOUNDARY LAYER WITH STRONG BLOWING

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The authors present a method of calculating nonsimilarity laminar boundary layer flow over a permeable flat plate with uniform blowing.

As was shown in [1, 2], the boundary-layer equations for zero-gradient flow over a permeable flat plate can be solved for a finite range of variation of the blowing parameter F_w. As F_w tends to a critical value the boundary layer thickness increases without bound, while the friction coefficient tends to zero. For the case of uniform blowing along the length of the plate similar conclusions were reached in [3, 4], from direct numerical solution of the boundary layer equations. However, the results of the experimental investigation of [5] indicate that the laminar flow regime can exist for large enough intensities of transverse mass flow. It was shown in [5, 6] that as the blowing parameter increases there is a gradual deformation of the velocity profile from the Blasius to a sharply pronounced S-shape typical of jet type flows. When F_w reaches the critical value one does not observe a sharp increase of the boundary layer thickness nor a change of the flow regime, i.e., the boundary layer separates smoothly from the wall. A simple analytical solution of [7] gives good agreement with the experimental data at moderate blowing intensities, as was shown in [6]. The unsatisfactory agreement between the theoretical and experimental velocity distributions with strong blowing is due primarily to the negative pressure gradient induced by the transverse mass flow, which is not accounted for in either the numerical solutions [3, 4] or the analytical solution [7].

1. To establish (i.e., find) the velocity distribution in the unperturbed boundary layer with uniform blowing, we use the results of an asymptotic analysis of the equations of motion employed in [8, 9]. These papers obtained the result that for strong blowing the dividing streamline characterizing the zero value of the stream function is a straight line, and the region bounded by it has the shape of the wedge

$$y_0 = (\pi^2 M^2/2)^{1/3} x. \tag{1}$$

Therefore, to describe the velocity distribution in the boundary layer with uniform blowing and allowing for the induced pressure gradient, we use the similarity family of Falkner–Skan profiles appropriate to flow over a permeable wedge with semiopening angle

$$f''' + f''f + \beta(1 - f'^2) = 0,\ \eta = 0,\ f = -f_w,$$
$$f' = 0;\ \eta = \infty,\ f' = 1. \tag{2}$$

Thus, it is assumed that the influence of blowing on the external flow may be an effective method of replacing the original problem by an equivalent one: flow over a body whose profile is formed by the dividing streamline between the blown gas and the incident flow. Then, assuming, on the basis of Eq. (1), that the blowing creates the same pressure gradient as in flow over a wedge, i.e., that these two flows are similar, we obtain the following relation between the pressure parameter and the blowing intensity:

$$\beta = \frac{2}{\pi} \operatorname{arctg} \left(\frac{\pi^2}{2} M^2 \right)^{1/3}. \tag{3}$$

We used Eq. (3) to calculate the velocity distributions shown in Fig. 1. In a comparison with the experimental data of [5], obtained for a Reynolds number of $Re_x = 5\cdot 10^3$, we find satisfactory agreement of the results for all intensities of transverse mass blowing.

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